The Brickfields Trust

Progression in Calculation

Kings Road Primary School

Lawford Mead Primary & Nursery School

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Next review date: Autumn 2025

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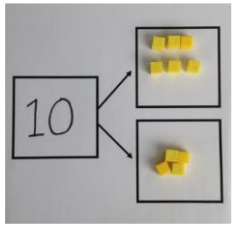
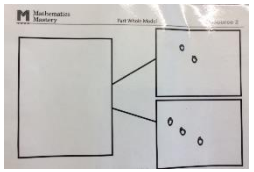
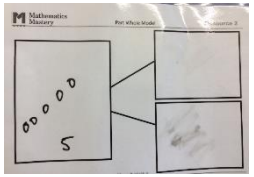
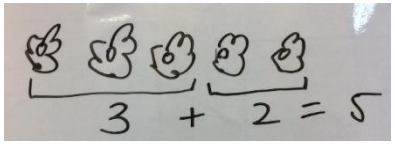
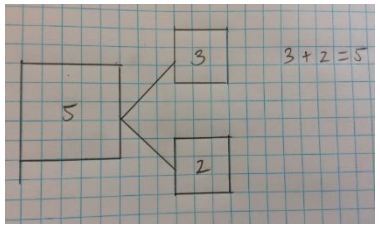
Progression in Calculations

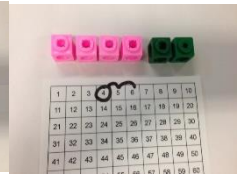
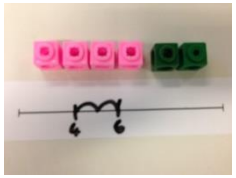
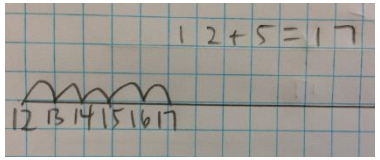
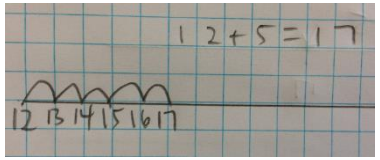
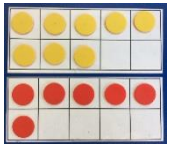
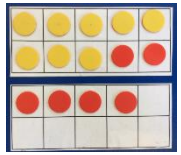
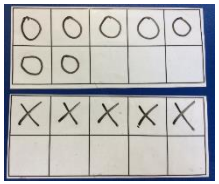
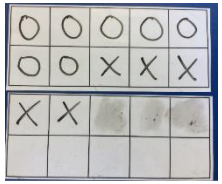
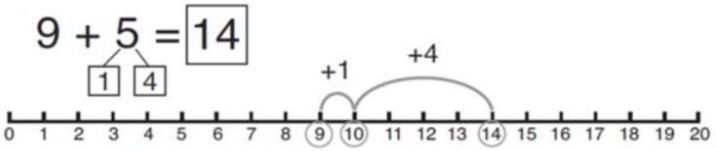
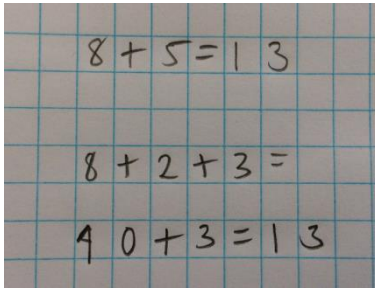
Progression of models and images Each year group to use the previous year groups' representations alongside their own age related expectations.			
EYFS The children should be provided with these models and images to use alongside manipulatives. tens frame cubes Numicon sorting hoops	Year 1 The children should be provided with these models and images to use alongside manipulatives. As for EYFS and… part whole model – all four operations Numicon cubes in sticks of 10 bead strings 1-20 place value chart - TO	Year 2 The children should be given these models and images to work on and taught how to draw them to represent them themselves during independent learning. As for EYFS and Year 1… arrays part whole model Base 10 bead strings 1-100 empty number line one hundred square place value chart - TO	KS2 Use all the previous models to bridge gaps in learning and then introduce… place value chart – relevant to year group Gattegno chart bar models fraction wall arrays

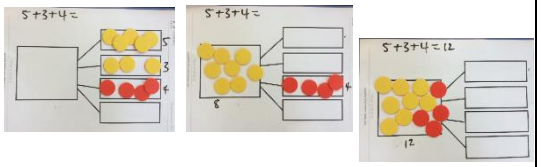
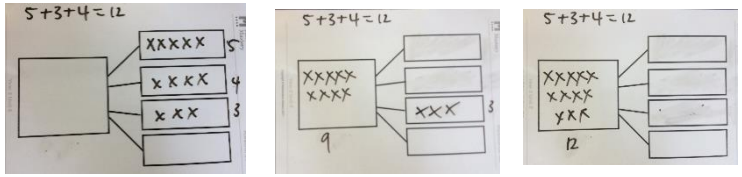
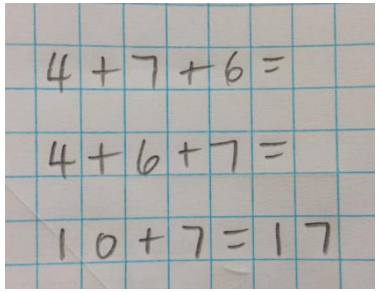
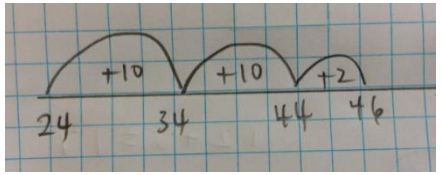
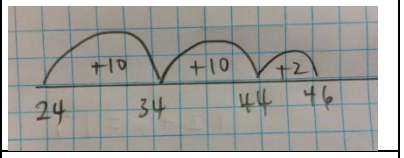
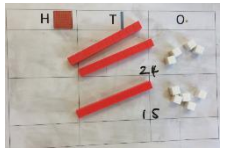
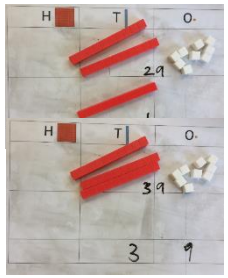
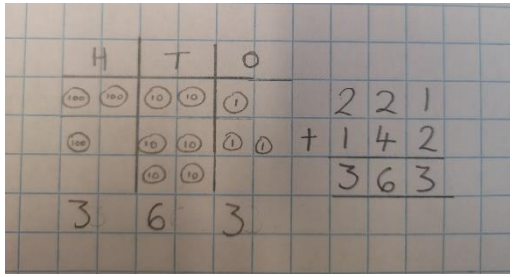
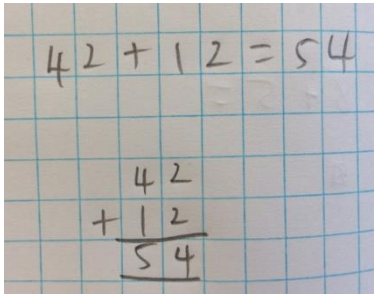
Addition

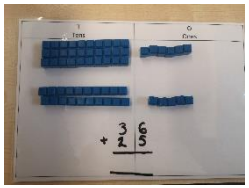
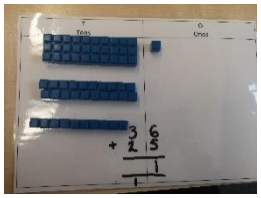
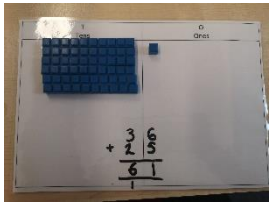
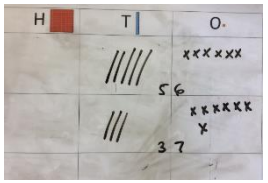
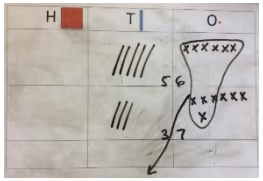
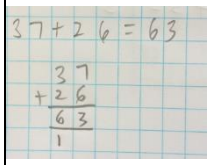
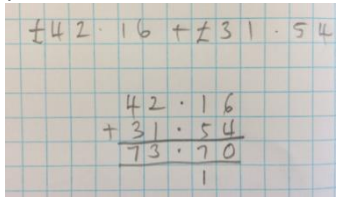
Key vocabulary:
calculate count equal more add addition sum addend altogether all in all together total plus number exchange regrouping place value is equal to part whole known fact partition

Key models, images and resources:
tens frame part whole model bar model column addition number line cubes Numicon bead strings Dienes place value counters

Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model	 Use resources to add two numbers together as a group – cubes, Numicon, Dienes etc. The ten frame is a strong structure for the children to use here.	Use pictures to add two numbers together as a group or represent as a bar or other useful image.   	 Use the part-part whole diagram as shown above to move into the abstract. This model also helps to show that 3+2=5/ 2+3=5/ 5=3+2/ 5=2+3

Starting at the bigger number and counting on	  Start with the larger number on the number line or number square and then count on the smaller number 1 at a time to find the answer.	Start at the larger number on the number line and count on in ones or in one jump to find the answer. 	Place the larger number in your head and count on the smaller number to find your answer. Children might count on in 1s still at this point. 
Regrouping to make 10.	Number bonds to ten are important in this stage.  	Use pictures or a number line. Regroup or partition the smaller number to make 10.   	 If I am at eight, how many more do I need to make 10? How many more do I add on now?

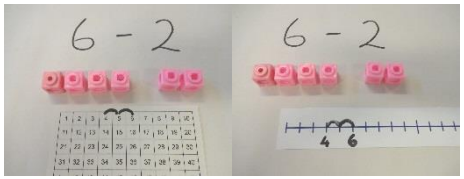
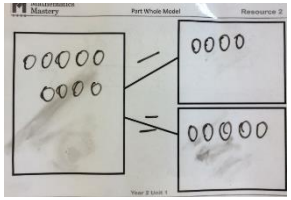
Adding three single digits	5 + 4 + 3= 12 Where possible find bonds to 10 first, then add the third digit. 	 Add together the three groups of objects, again looking for the bonds to 10 where possible. A picture could be drawn to represent the three numbers also.	 Combine the two numbers that make 10 and then add on the remainder.
Adding on a number line.	This will require the children to jump on a number line. They will identify the starting number (the biggest) and jump in appropriate steps for the year group.	Children will draw their own number line. 	Children will begin to add numbers by making jumps. But these will be in their head as follows. 24+22= 22+10+10+2= 
Column method- no regrouping/ exchanging	  Represent each number with Numicon. Add the ones and then add the tens. Or could use the Base 10.	Children might also draw the resource in place value columns. This could include drawing the Base 10 or the place value counters. The abtract representation could be alongside this. 	This then becomes the more formal column addition method: 

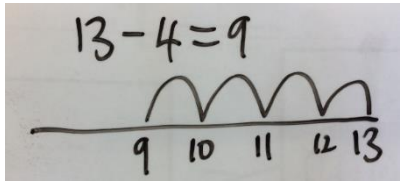
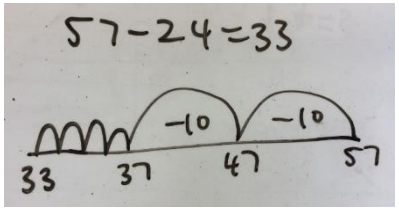
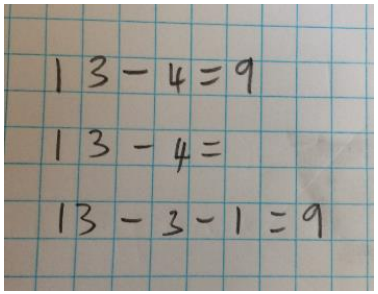
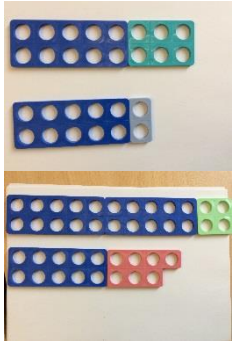
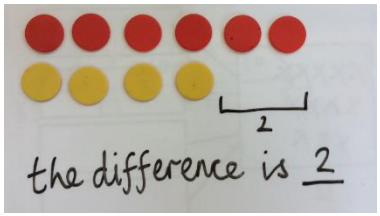
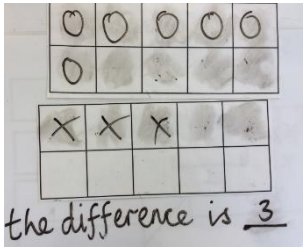
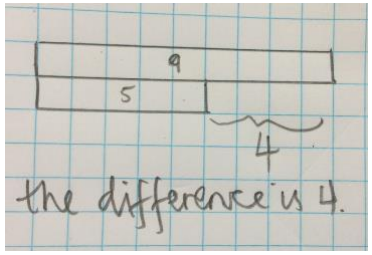
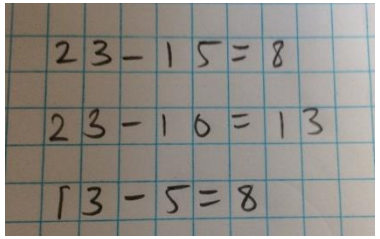
Column method- regrouping/ exchanging	 Represent the two numbers being added on a place value chart. Add together the ones and exchange/ regroup where necessary.   Add together the tens – not forgetting any that were exchanged from the ones addition. Continue for other place value columns.	  56+37= The children can then see the exchanging of ten 1s into one 10 and therefore understand why the 10 is moved into the tens column.	Start by partitioning the numbers before moving on to clearly show the exchange below the addition.  This strategy can be used for decimal numbers too. It is essential the children line the place value columns correctly. 

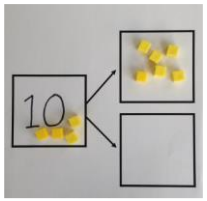
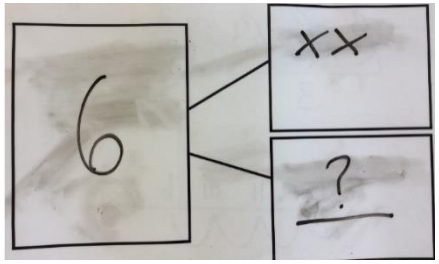
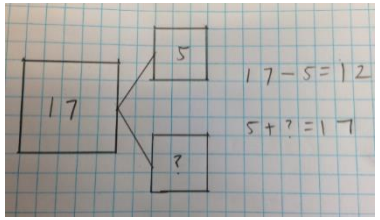
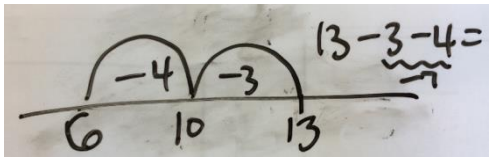
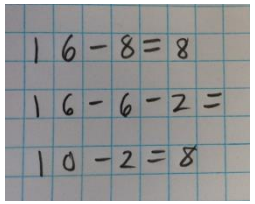
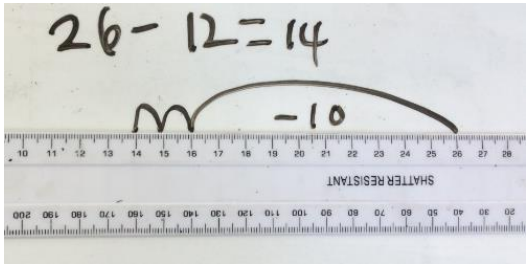
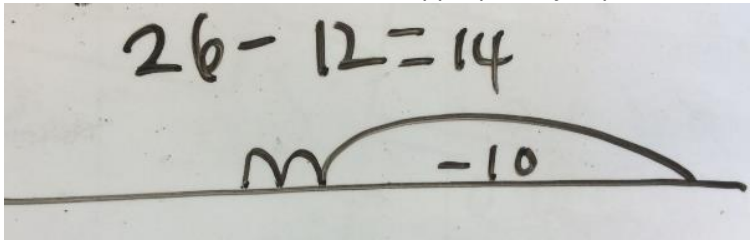
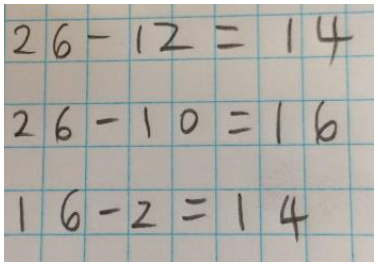
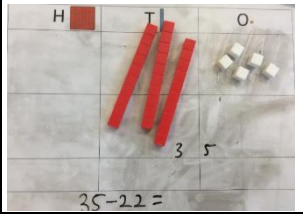
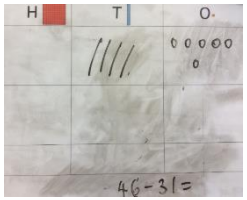
Subtraction

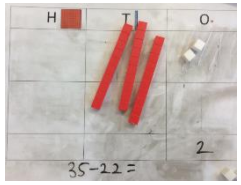
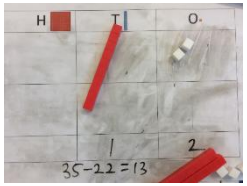
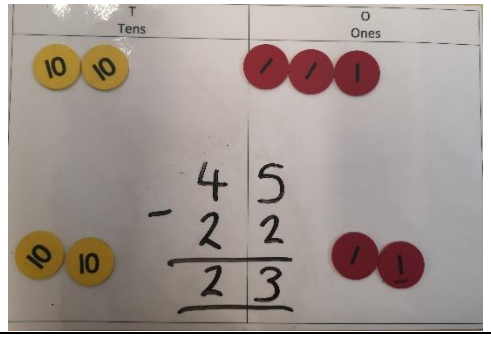
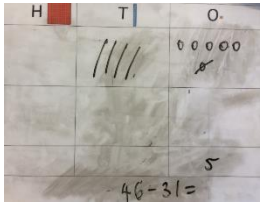
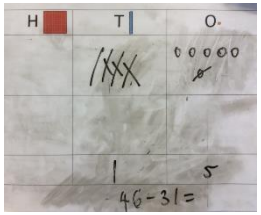
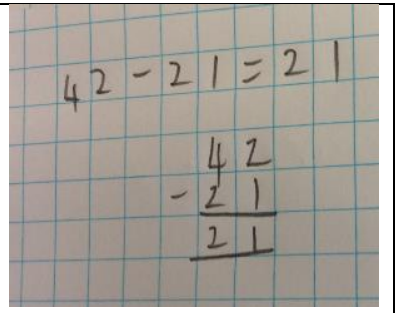
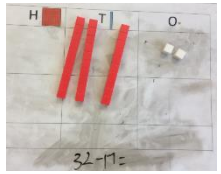
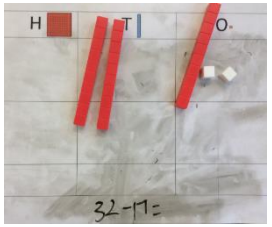
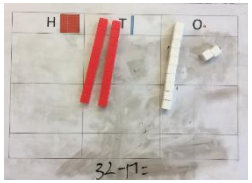
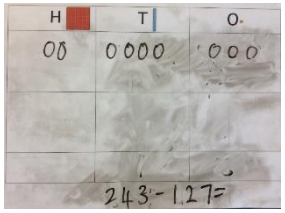
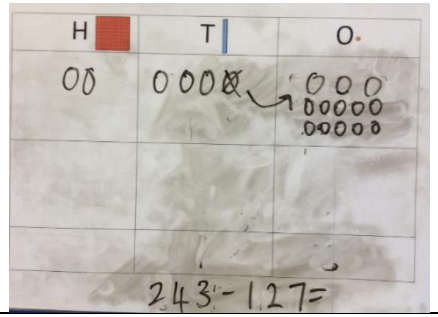
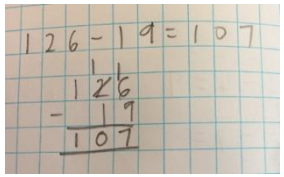
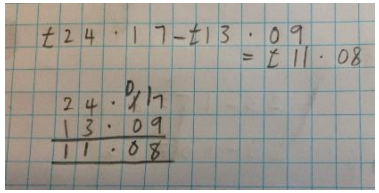
Key vocabulary:
calculate count equal fewer less subtract subtraction minus more than greater than take away fewer than less than difference subtrahend minuend How many left? How many less? number exchange regrouping place value is equal to part whole known fact partition

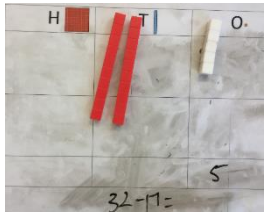
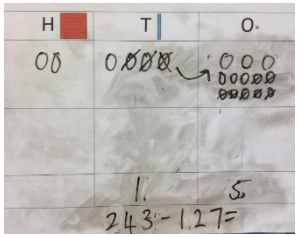
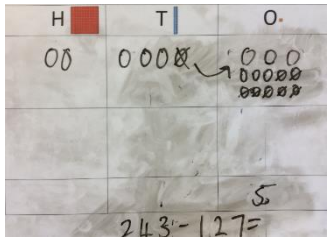
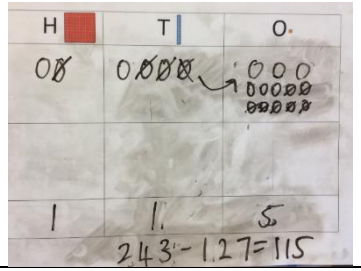
Key models, images and resources:
tens frame part whole model bar model column subtraction number line cubes Numicon bead strings Dienes place value counters

Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, Unifix etc. to show how objects can be taken away. 6 – 2 = 4  	Cross out drawn objects to show what has been taken away.  	18 -3= 15 8 – 2 = 6 The children will be counting back in ones.

Counting back	13 – 4: Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. 	Start at the bigger number and count back the smaller number showing the jumps on the number line.   This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.  Children can begin to use partitioning to speed up this step e.g. 13-4= (13-3)-1
Find the difference	Compare amounts and objects to find the difference. Use Numicon to create the numbers and lie them alongside each other to see the difference between them. 	 Count on to find the difference. Use representations where children can compare the value of the numbers.   Draw bars to find the difference between 2 numbers.	The children will securely understand that you can subtract to find the difference. 

Part Part Whole Model	 This model helps make the link with addition explicit. If 10 is the whole and 6 is one of the parts. What is the other part?	 Use a pictorial representation of objects to show the part part whole model.	 Move to using numbers within the part whole model.
Make 10	Make 14 on the ten frame. Take away the 4 first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 – 8= Subtract ones to the next ten. How many do we have left to take off? 
Number line	Children to use actual number lines or hundred squares to calculate the counting back. 	Draw their own number line with appropriate jumps. 	Children to complete the 'jumps' in their head when counting back. 
Column method without exchanging/ regrouping	 Use Base 10 create the number on a place value chart.	 Very similar to concrete, however the children draw their representstions on place value charts instead.	This will lead to a clear written column subtraction.

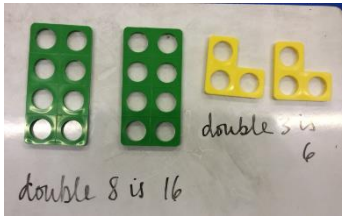
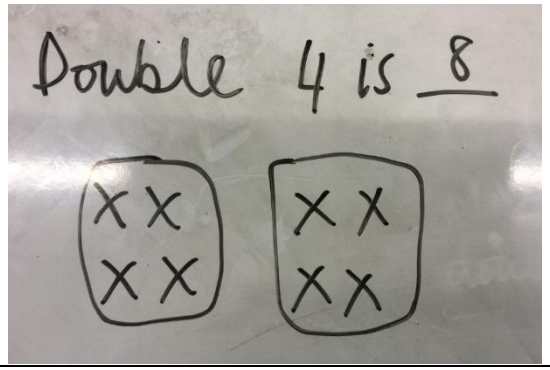
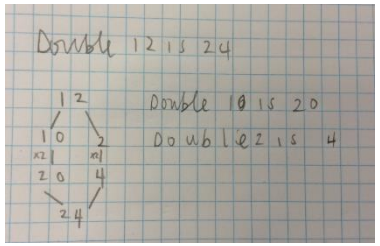
	Subtract the ones by taking them away.   Subtract the tens by taking them away. This leaves you with the answer.  	Cross out the ones to subtract them.   Cross out the tens to subtract them. This process should be continued with larger numbers (Tens → hundreds→ thousands…).	
Column method with exchanging/ regrouping	 Use Base 10 to start with. A ten is taken from the tens column and placed in the ones column.   These are exchanged into ones.	 Draw the Base 10 onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.  One ten is exchanged into ten ones. This is done by crossing a ten out and drawing ten more ones.	 

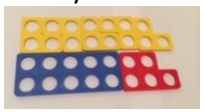
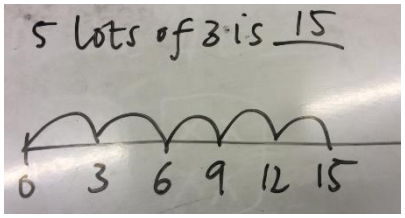
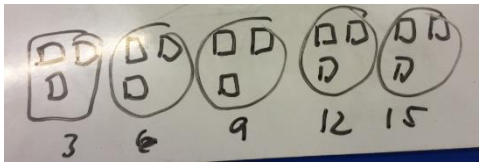
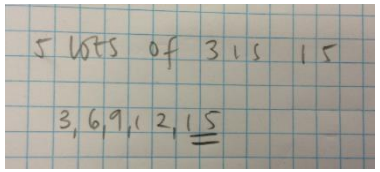
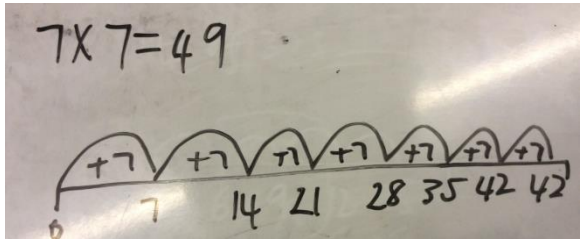
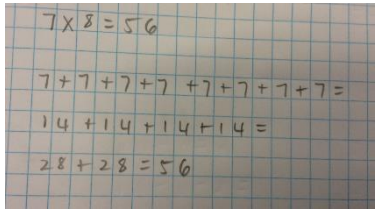
	Subtract the ones by taking them away from the representation.  Subtract the tens by taking them away from the representation. Continue for other place value columns.	Subtract the ones by crossing them out.  Do the same for the tens. Cross out the hundreds in the same way.   	

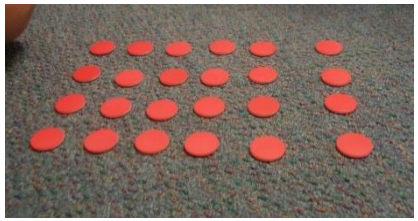
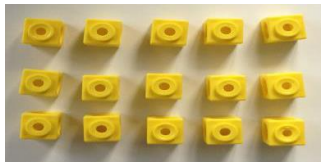
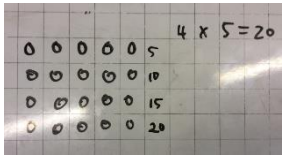
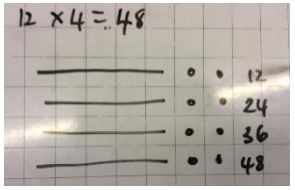
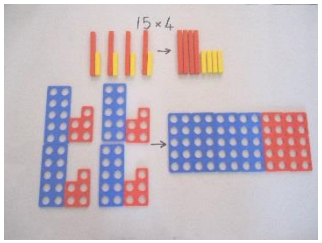
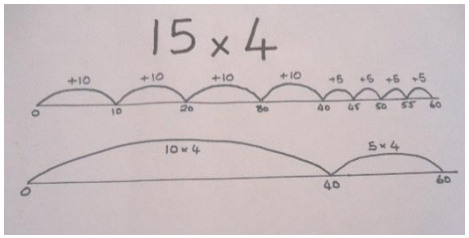
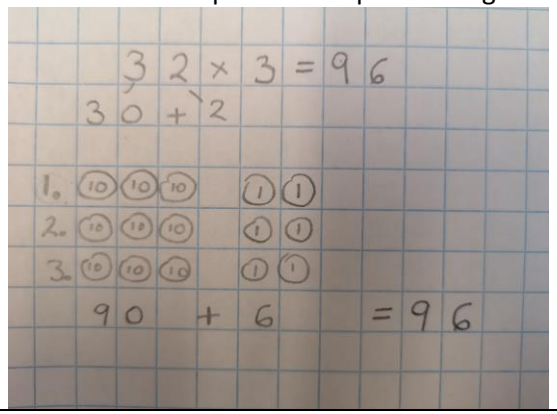
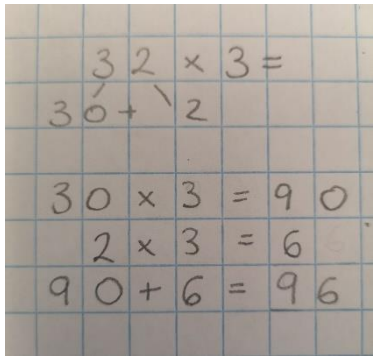
Multiplication

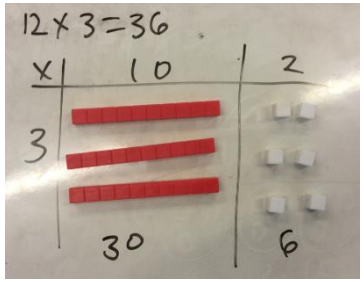
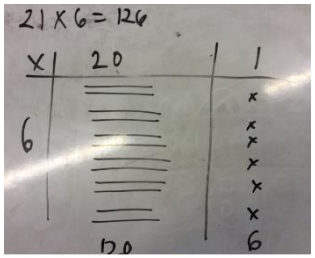
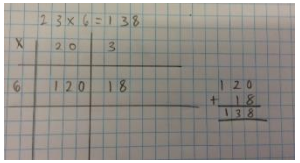
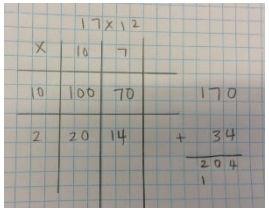
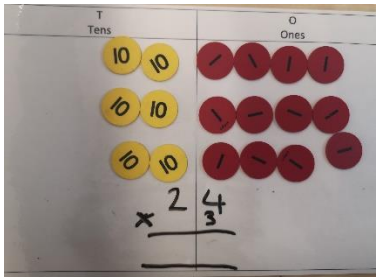
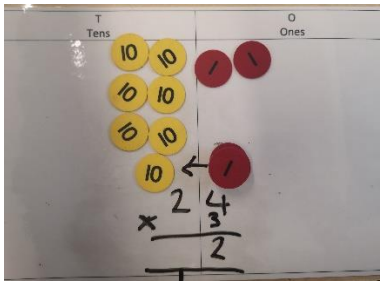
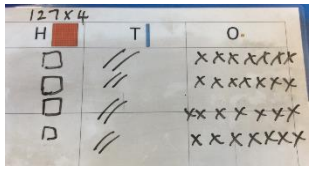
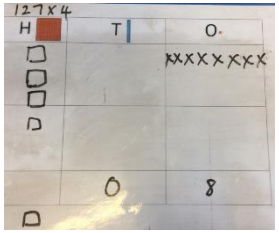
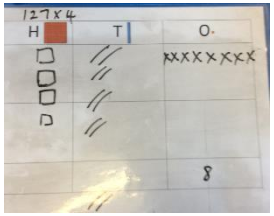
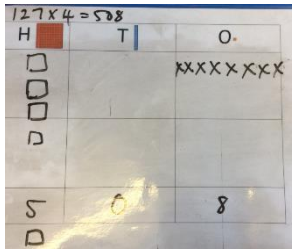
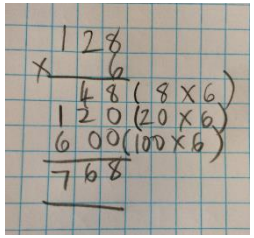
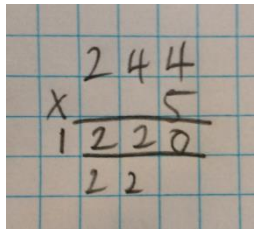
Key vocabulary:
multiply multiplication multiple of times product factor lots of groups of multiplied by repeated addition number exchange regrouping place value is equal to part whole known fact partition long multiplication short multiplication

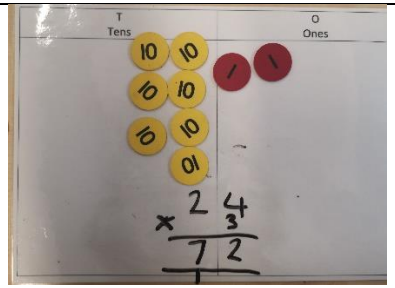
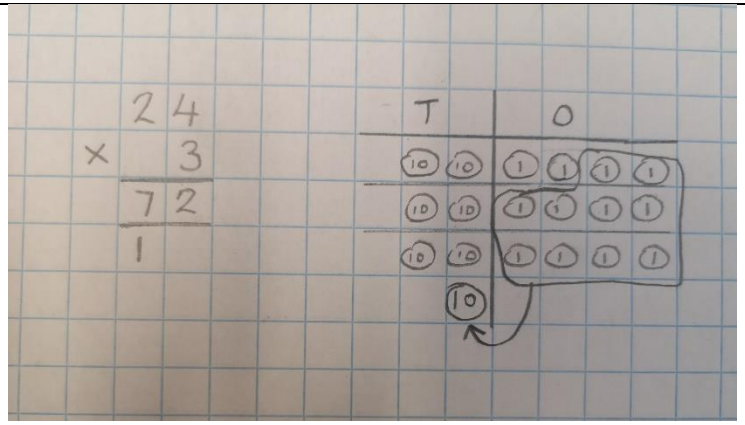
Key models, images and resources:
arrays grid method part whole model bar model grid method partition column multiplication number line cubes Numicon bead strings Dienes place value counters

Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number. This could include Numicon, cubes, Base 10 or any other countable objects.  	Draw pictures to show how to double a number. Double 4 is 8.  	Partition a number and then double each part before recombining it back together. 

Counting in multiples – skip counting	Count in multiples supported by concrete  objects in equal groups. The Numicon will show the multiples and they can be easily repositioned to count up easily. A number square, number line or number track could help too. 	Use a number line or pictures to continue support in counting in multiples.   	Count in multiples of a number aloud. Write sequences with multiples of numbers. 
Repeated addition	 Similar to counting in multiples however, this moves onto the children choosing a wider range of objects to use to repeatedly add the same amount.	The children will represent the multiplication so that they can repeatedly add the same amount the correct number of times. This could be on a number line.  	Write addition sentences to describe objects and pictures. 

Arrays- showing commutative multiplication	Create arrays using counters/ Unifix to show multiplication sentences.  	 Draw arrays in different rotations to find **commutative** multiplication sentences . Children will then use more efficient ways to represent the grid method pictorially.  Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition. As well as the beginning to have an understanding of the inverse relationship between multiplication and division.
Grouping on number line or using partitioning to solve	Children will begin to partition the number being multiplied and solve it as shown below.  	Children to use their knowledge of representing repeated addition on a numberline. They will begin to see that they can complete more effiecient jumps:  You could also represent the partitioning method like this:  	Children to be able to see a multiplication as follows: 15x4 = (10x4) +(5x4) 40+20=60 Another way to represent: 

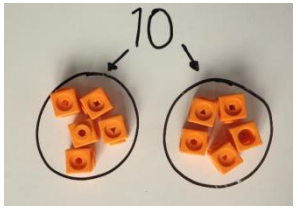
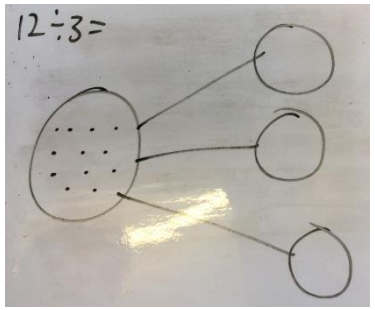
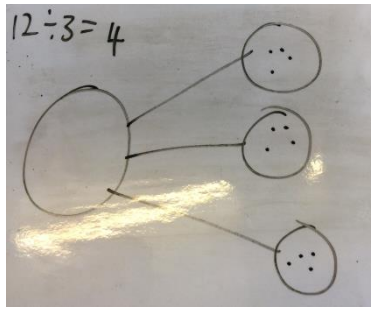
Grid Method	Show the link with arrays to first introduce the grid method. Move on to using Base 10 to move towards a more compact method.  	Children can represent the work they have done by drawing Base 10 (or any other resource such as place value counters) in a way that they understand.  	 
Column multiplication	Create the number being multiplied the correct number of times within the place value columns. Then combine the ones, exchange if needed. Then combine the tens, exchange if needed. And so on.   	 Exchanges can be shown by rubbing out the ones and drawing the tens. Or by crossing out the exchanged ones.  Very similar to concrete, however the Base 10 or place value counters are drawn instead.   	 This is a good sub-step to scaffold the formal efficient multiplication method. 

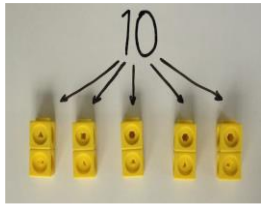
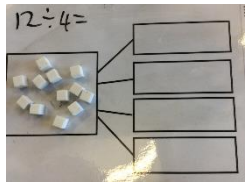
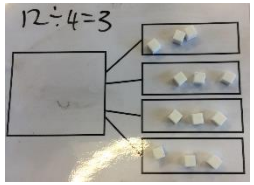
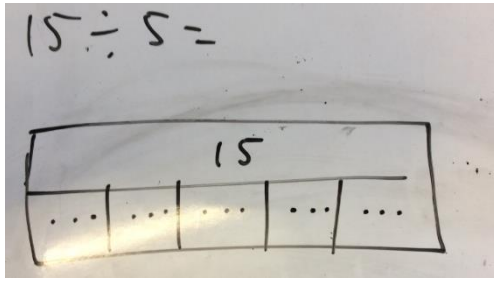
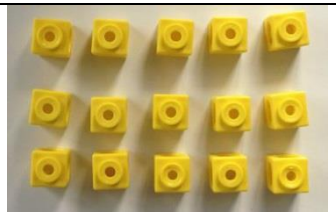
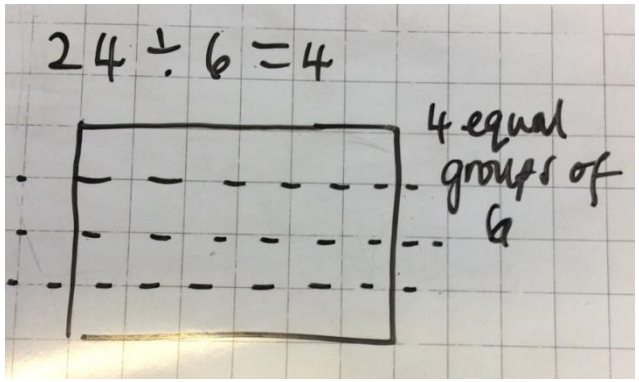
	 T Tens O Ones 10 10 10 10 10 10 10 1 1 24 × 3 72	 24 × 3 72 1 T O 10 10 1 1 1 1 10 10 1 1 1 1 10 10 1 1 1 1 10	

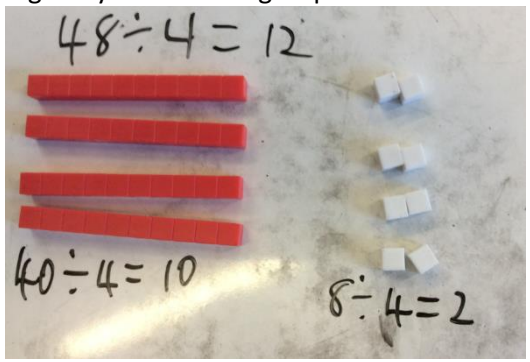
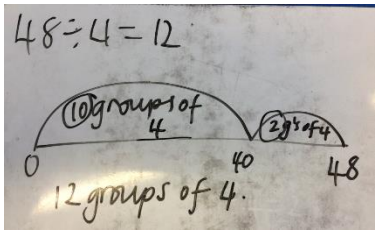
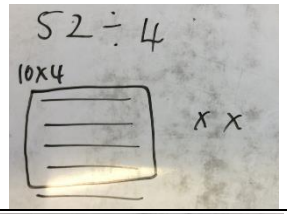
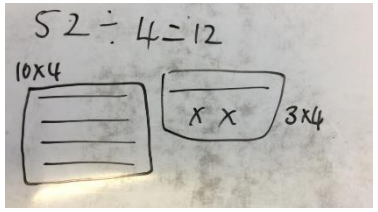
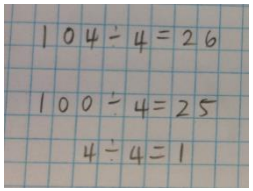
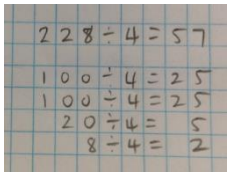
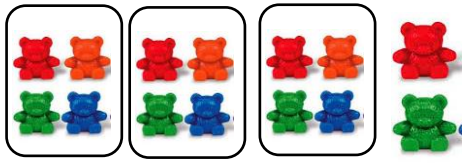
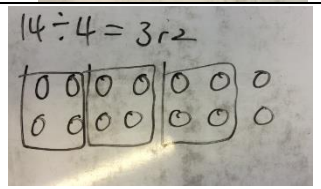
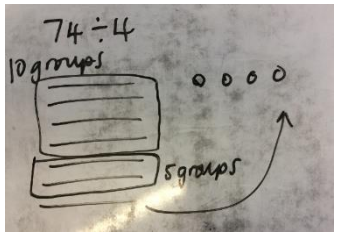
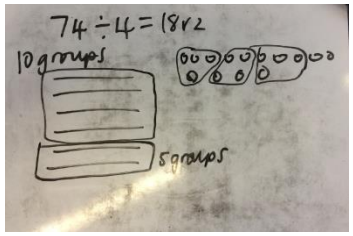
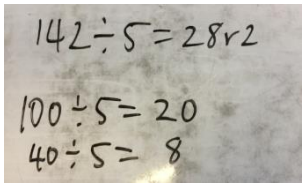
Division

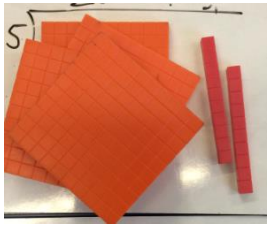
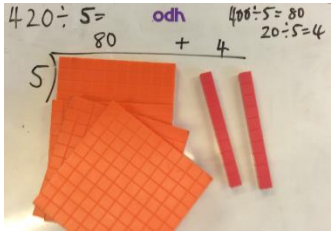
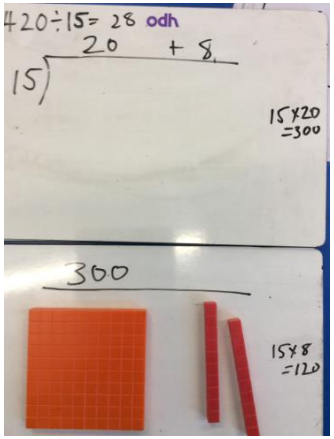
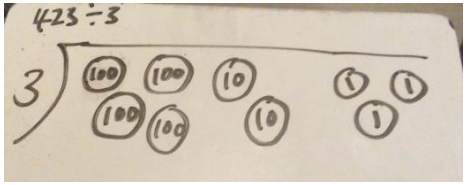
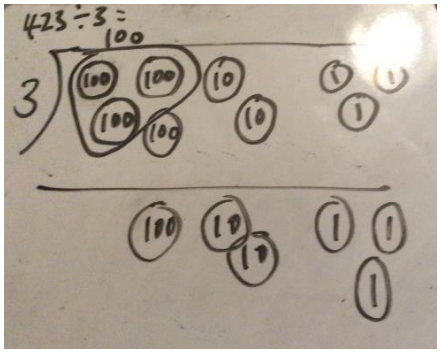
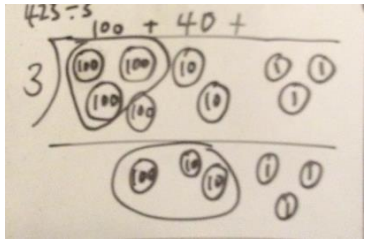
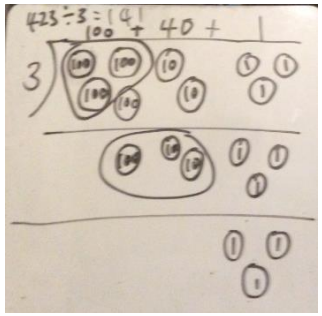
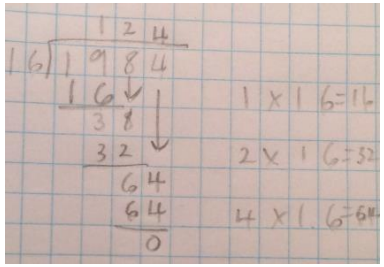
Key vocabulary:
division divide share half divided into equal groups of share equally remainder repeated subtraction number exchange place value is equal to part whole known fact partition dividend divisor quotient short division long division divisible

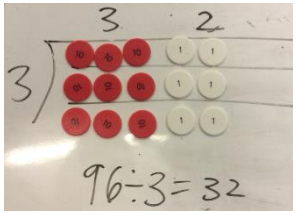
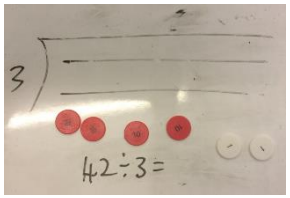
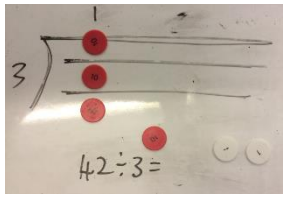
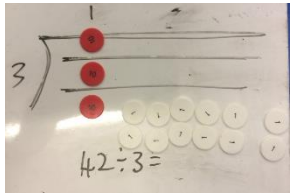
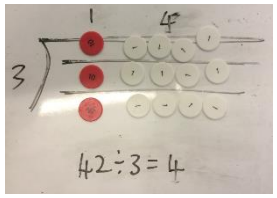
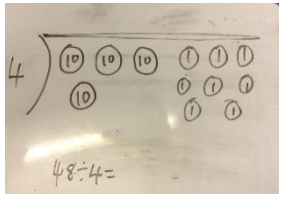
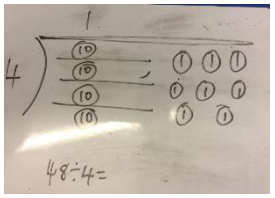
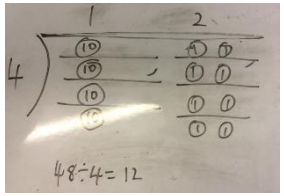
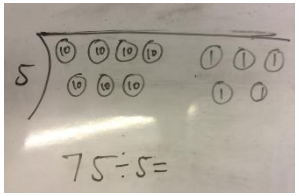
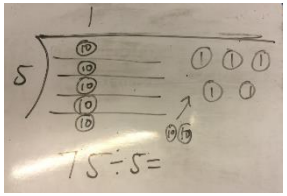
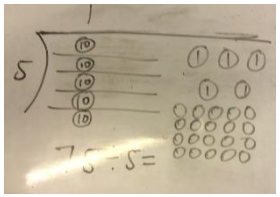
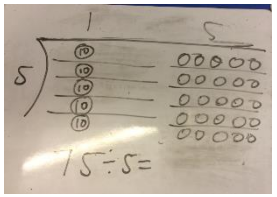
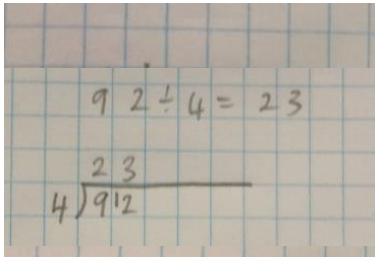
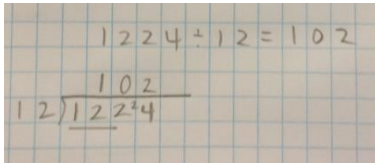
Key models, images and resources:
equal grouping sharing equally tens frame part whole model bar model number line cubes Numicon bead strings Dienes place value counters

Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	children to physically share items into equal groups. This could be using a part whole model. This shows 10÷2=5 	Children use pictures or shapes to share quantities.  	Share 9 buns between three people. 9 ÷ 3 = 3 Children begin to become confident and use times table knowledge at this point. Or they count in 3s to find how many 3s are in 9.

Division as grouping	 Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  A part whole mode is also useful here. 	Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?
Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences. 	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7

Division using bigger groupings.	Children begin to see how to represent large amounts of the equal groups. e.g. they can see the groups of four in 48 	 number line notation works well for this skill. Base 10 representations or place value counters also enable the children to see larger groups.  	Children can begin to partition to find the equal groups.  
Division with a remainder	Divide objects between groups and see how many are left over. 14 ÷ 3 = 	 Children's pictorial represnetations will enable them to see remainders clearly. This leads to children being able to identify larger equal groups and then see if there is a remainder.  	Complete written divisions and show the remainder using r. 29÷8=3 r5 <u>8, 16, 24,</u> 25, 26, 27, 28, 29 Children will also be able to partition into larger numbers of equal groups. 

Long division	 Initially represent the calcualtion with a suitable resource i.e. Base 10. This shows 420 ÷5. Children to use their knowledge of times table to know how many 5s are in 400.  They then use their knowledge of times tbale to know how many 5s are in 20.  This can also be done with extending notations where the children use times table knowledge they are confident with.	 Represent the division. Use a known fact for the hundreds. How many 3s are in 300. Bring any undivided hundreds down.   Complete the same with the tens and remaining hundreds. Finally, the same with the ones.  	It is important that the children understand that by starting at the hundred and saying that there is one 16 in 19. That they are clear that it is 100 groups of 16 in 1, 900.  Also, the 3 remaining is 3 hundreds and not just '3'.

Short division	 Represent the division with place value counters. Share the tens into the correct number of groups – in this case 3 equal groups. This can be linked to 90÷3=30 so we put a 3 in the tens place. Then complete the same step for the ones.  Create the division alonside the calculation. Share the tens into equal groups.   Move any tens into the ones column, and exchange them. Share the ones into equal groups. This create your answer. 	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Draw the division within the layout. This should look similar to the concrete resources used.  Share the tens into equal groups. Write how many are in each group.  Do the same fot the ones. This method should be used with divisions that work out nicely at first. When it doesn't work out nicely. Follow the same strategy, but be prepared to exchange as you need to.    	resources used When solving problems using the abstract method, the children need to be able to reason what is happening. Examples such as the one below should be given at first.  Then one exchange should be provided. Before dividing by a two-digit number is given.